

Zadání zápočtových a zkuškových písemek z limit v kurzu Diferenciální počet na PedF UK z let 2005/6 a 2006/7:

A. Bez užití L'Hospitalova pravidla a Taylorových polynomů spočtete limitu:

1.  $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 4x + 7} - \sqrt{9 - 3x}}{2x^3 - 9x^2 + 12x - 4}$
2.  $\lim_{x \rightarrow -\infty} \left( \sqrt{4x^2 - 3} + 2x \right) \ln(x^4 + 2^x)$
3.  $\lim_{x \rightarrow -3} \frac{x^3 - 7x + 6}{(\sqrt{x^2 + 6x + 10} - \sqrt{2x + 7})^2}$
4.  $\lim_{x \rightarrow -\infty} \frac{1 - \cos \frac{1}{x}}{(3x + \sqrt{9x^2 - 2})^2}$
5.  $\lim_{x \rightarrow \infty} \frac{\ln\left(\frac{2}{\pi} \arctg x\right)}{\sqrt{4x^2 + 4x + 1} - 2\sqrt{x^2 + x}}$
6.  $\lim_{x \rightarrow 0+} e^{x - \ln x} (\cos \sqrt{3^x - 1} - 1)$
7.  $\lim_{x \rightarrow \infty} \ln \left( \frac{2x^2 - x + 1}{2x^2 + x - 1} \right) (x + \sin x)$
8.  $\lim_{x \rightarrow 2} \frac{\ln(x^2 - 3)}{x^6 - 12x^4 + 48x^2 - 64}$
9.  $\lim_{x \rightarrow \infty} \frac{\operatorname{arccotg} \frac{x^4}{1+x^2}}{\cos \frac{x}{x^2+1} - 1}$
10.  $\lim_{x \rightarrow 0} (\cos x)^{\operatorname{arccotg}^{-1} x^{-2}}$
11.  $\lim_{x \rightarrow 1} \frac{3}{x^3 - 1} - \frac{5}{x^5 - 1}$
12.  $\lim_{x \rightarrow 2} \frac{\ln \frac{4^x}{2x^2}}{x^3 - 2x^2 + x - 2}$
13.  $\lim_{x \rightarrow \infty} \ln^2 x^2 - \ln^2(x^2 + 1)$
14.  $\lim_{x \rightarrow \infty} \frac{\ln x - \ln \sqrt{x^2 + 1}}{\operatorname{arccotg} x}$
15.  $\lim_{x \rightarrow 0} \frac{\ln \left( \frac{1 + \cos x}{2} \right)}{e^{x^2} - 1}$
16.  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{\operatorname{tg} x - \sin x}$
17.  $\lim_{x \rightarrow -1} \frac{1 - \sqrt{3 + x - x^2}}{\ln(x^5 + 10x^2 + 15x + 7)}$
18.  $\lim_{x \rightarrow 0} \frac{\arccos e^{-x^2}}{x}$
19.  $\lim_{x \rightarrow 0} \frac{\ln(\cos x + \sin x) + \ln(\cos x - \sin x)}{\sqrt{x^3 + 2x^2 + 2x + 1} - x - 1}$
20.  $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{\operatorname{tg}(\sin(\operatorname{arctg} x^3))}$
21.  $\lim_{x \rightarrow 0} \frac{\ln(\sin x - \cos x)^2}{\sqrt{1 - \cos x}}$
22.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + 2x - x} \right)^x$
23.  $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 + \operatorname{tg} x}}{\operatorname{arctg} x^3}$
24.  $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin^2 x} - \cos x}{\ln \cos x}$
25.  $\lim_{x \rightarrow 2} \frac{\sqrt{1 - \cos(\pi x)}}{\sin(3x^3 - 10x^2 + 4x + 9)}$
26.  $\lim_{x \rightarrow \infty} (e^{-x} + \operatorname{arccotg} x) \ln(x^{\sqrt{x}} + x^2)$
27.  $\lim_{x \rightarrow 1} \frac{\ln x}{\arccos \frac{2x}{x^2+1}}$
28.  $\lim_{x \rightarrow -1} \frac{2^{-x} - 2}{\ln(x - 3\sqrt[3]{x} - 1)}$
29.  $\lim_{x \rightarrow -\infty} x \sin \frac{\operatorname{arctg}^2 \frac{1}{x}}{\operatorname{arccotg} x - \pi}$
30.  $\lim_{x \rightarrow 0} (\cos x)^{(1 - \sqrt{1 - \sin^2 x})^{-1}}$
31.  $\lim_{x \rightarrow \infty} x \arccos \frac{2 \operatorname{arctg} x^2}{\pi}$
32.  $\lim_{x \rightarrow 1} \frac{2^x - 2\sqrt{2^x - 1}}{\ln(2x^5 - 5x^4 + 10x^2 - 10x + 4)}$
33.  $\lim_{x \rightarrow 1} \frac{\sqrt{\ln^2 x + 1} - \ln x - 1}{\sin(2\pi x)}$

B. Bez užití L'Hospitalova pravidla a Taylorových polynomů spočtete limity funkce  $f$  v krajních bodech  $\mathcal{D}(f)$ :

1.  $f(x) = \frac{\sqrt{x^2 + 1}}{x \operatorname{arctg}(2^x - 1)}$
2.  $f(x) = \frac{\sqrt{\ln(1 + x)}}{\ln(1 + \sqrt{x})}$
3.  $f(x) = \frac{\sin e^x}{e^x - e^{2x}}$
4.  $f(x) = \frac{\ln(e^x - x) - x}{\operatorname{arctg} x}$
5.  $f(x) = \frac{xe^x}{\sqrt{e^{x^2} - 1}}$
6.  $f(x) = \frac{3^x \sin \frac{1}{x}}{e^x - \ln x}$
7.  $f(x) = \frac{\operatorname{arctg}(x e^{-x})(e^x + 1)}{\sqrt{x^2 - x + 1} - x - 1}$
8.  $f(x) = \frac{\sin \frac{x^4}{x^2+1}}{\ln \frac{x^4+x^2+1}{x^2+1}}$

9.  $f(x) = \frac{\sqrt{x^2+1} + x - 1}{e^{2x} + e^x - 2}$
10.  $f(x) = \frac{e^x \sin x}{x^x - 1}$
11.  $f(x) = \frac{\operatorname{arctg} x}{(x + \sin x) \operatorname{arccotg} x}$
12.  $f(x) = \frac{\operatorname{arctg} x^2}{(\sqrt{x^2 + 2x + 4} - x - 2)^2}$
13.  $f(x) = \frac{\sin(x + \sqrt{x^2 + 1})}{e^x}$
14.  $f(x) = \frac{\operatorname{arctg} x \operatorname{arccotg} x}{\sqrt{x^2 + x + 1} - x - 1}$
15.  $f(x) = e^x \sin(x \operatorname{arccotg}^2 x)$
16.  $f(x) = \frac{\sin((\pi - \operatorname{arccotg} x) \sqrt[3]{x})}{e^{e^x} - 1}$
17.  $f(x) = \frac{\sin(x^2 e^{-x})}{1 - \sqrt{x^2 + 1}}$
18.  $f(x) = \operatorname{arccotg}(-x) \cdot \sin \frac{\sqrt{x^2 + 1} - 1}{(e^x - 2^x)^2}$
19.  $f(x) = \frac{1 - \cos \sqrt{x}}{\ln \frac{x+1}{x^2+1}}$
20.  $f(x) = \frac{\cos e^x - 1}{3^x - 2^x}$
21.  $f(x) = \frac{e^{\sqrt{x^2+1}-x} - 1}{(e^{-x} + 1) \operatorname{arccotg} x}$
22.  $f(x) = \frac{\sin(\sqrt{x^2 + 2} - x - 1)}{\operatorname{arccotg} x}$
23.  $f(x) = \frac{\sqrt{2 - \cos x} - 1}{e^{x^2} - 1}$
24.  $f(x) = \frac{\sin(\cos x - 1)}{\ln \frac{x^4+1}{x^2+1}}$
25.  $f(x) = \frac{x}{(\sqrt{x^2 + x + 1} + x)^2} \ln \frac{\operatorname{arccotg} x}{\pi}$
26.  $f(x) = \frac{\sin(x \operatorname{arccotg} x)(\sqrt{x^2 + x + 1} + x)}{\operatorname{arctg} x}$
27.  $f(x) = \frac{(\sqrt{\ln^2 x + x} + \ln x) \ln x}{x}$
28.  $f(x) = \sin(x \operatorname{arccotg} x^2) \sqrt{x^2 + \frac{1}{x^2}}$
29.  $f(x) = \frac{\sin \ln \frac{x^2}{x^2+1}}{(e^{\frac{1}{x}} - 1)^2}$